Essential work of fracture in polymer films

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The fracture behaviour of two polymer films (100 μm thick) has been investigated according to the essential work of fracture (*EWF*) approach in order to assess the validity of the linear relationship between the specific total work of fracture and the ligament length, postulated to exist by the *EWF* theory, and examine the transition from plane strain to plane stress conditions. Double edge notched specimens loaded in tension were used; ligaments were varied in the range 0.3 to 45 mm. No linear relationship was found and it was confirmed that a power law can be very accurately fitted to the experimental data. An annealed copper film (100 μm thick) exhibited the same pattern of behaviour.

1. Introduction

The definition of parameters that identify the critical conditions for the propagation of fracture is of paramount importance in mechanical engineering. Such parameters are also very useful to materials scientists for the development of tougher and tougher materials. However, the simple identification of critical conditions (to which all fracture tests are devoted) is often of little help in elucidating the relevance of the various mechanisms of energy dissipation activated within the plastic zones. The essential work of fracture (EWF) approach starts just from a model for the development of the plastic zone around stress concentrators and seems to be better suited to understand the fracture behaviour of polymers. The idea, originally set forth by Broberg [1–3], is that the yielded zone that encircles the crack tip in a notched body can be divided in two parts, one enclosing the other. The energy required to propagate the crack is also split into two components: one, absorbed in the inner plastic region, close to the crack trajectory, is the Essential Work of Fracture, U_E; the other, dissipated in the outer region that surrounds the inner one, is the Non-Essential Work of Fracture, U_{NE}. The total work of propagation, U_{TOT}, is then $U_{TOT} = U_E + U_{NE}$. It is assumed that the essential term scales with the cross-section of the ligament (since the height of the inner zone is taken as a constant) and that the non-essential term scales with the volume, V_P, of the total plastic zone that, in turn, scales with the square of the ligament (that is $V_P = \beta l^2 t$, where β is a shape factor of the plastic region, l is the ligament size and t is the sample thickness). By referring the work of propagation to unit surface we have:

$$u_S = u_E + \beta l u_{NE} \quad (1)$$

where u_S ($= U_{TOT}/lt$), u_E, u_{NE} are the total specific, specific essential and specific non-essential works, respectively.

Two deformation regimes are supposed to exist, depending on the fraction of the size of the ligament to the sample thickness (l/t). At large ligaments, i.e. $l/t > 3 \div 5$, plane stress conditions prevail in the ligament and, according to Equation 1, a linear dependence between u_S and l should be observed. On decreasing the ratio l/t, plane strain conditions progressively take over and the work of propagation decreases. In such a mixed-mode regime, linearity no longer holds and the extrapolation of u_S data to $l = 0$ yields values of the specific essential work smaller than those obtained from the large ligament region.

It is apparent from the requisite $l/t > 3 \div 5$ than thin films should always be in the plane stress regime, irrespective of ligament length (for $t = 100$ μm it would suffice having $l > 0.3 \div 0.5$ mm) so that the relationship between u_S and l is expected to be linear in a very range of ligaments. To examine this point, and to assess the possibility of using the EWF approach to quantify the fracture resistance of thin sheets, we studied the behaviour of two polymers and of one metal. These materials showed very different post-yield behaviour; one polymer noticeably strain-softened, the other moderately strain-hardened and the metal markedly strain-hardened.

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2. Experimental details

Films 1 and 2 were common sheets for overhead projection; film 1 was a handwriting grade (by Staedtler) and film 2 was for copying machines (by 3M). Film 3 was annealed copper. An annealed grade was chosen in order to maximize the rate of hardening during the tensile tests. All films were 100 μm thick.

For *EWF* testing the samples were 75 mm wide and 120 mm long and had two identical notches on the opposite longest sides (DENT geometry). Notches were made with sharp scissors and their length was measured with an optical microscope before testing. Tensile properties were measured on rectangular strips, 10 mm wide and with a distance between the grips of 50 mm. Tests were carried out at room temperature with an Instron machine (type 4302); cross-head speed was 5 mm min^{-1}. Data were collected by computer and analysed with a spreadsheet.

3. Results

The stress extension curves in Fig. 1 indicate that the examined materials had very different post-yield behaviour. Film 1 had a uniaxial yield strength of about 45 MPa and exhibited, after yield, a large drop in stress (about 30%) due to softening. Film 2 was much stiffer and had a yield strength of about 95 MPa. Strain hardening set in short after yield and counterbalanced the strain softening so that the yield region was substantially flat. A stress increase was manifest at large deformations. Film 3, like many metals, showed a very pronounced strain hardening. The yield strength, about 80 MPa, and the ultimate strength are typical of commercial copper annealed to fine grain size.

Collections of load–extension curves of like specimens, only differing in ligament length, are reported in Fig. 2 for all materials. These curves reflect the tensile properties of the different films. For the two polymers the load was maximum on yielding the ligament, whereas for the metal yielding occurred much before the maximum and the load kept increasing beyond yield since the ligament strengthened up because of strain hardening. For this reason the plastic zone of the metal was much larger than that for the two polymers. The likeness, for each material, of the curves suggests that samples fractured under similar stress conditions.

3.1. Films 1 and 2

On stretching doubly notched samples, the load steadily rose and reached a maximum when the ligament yielded. After yield the strain softening, which characterizes most polymers, forced the deformation to remain localized (thickness locally decreased), whereas fracture started propagating from both notches; eventually sample separation occurred when the two opposite crack foreparts joined in the middle of the sample and the load vanished. The total work of fracture, U_T, obtained by integrating the individual load/extension curves is plotted, as a function of ligament length, in Fig. 3 for film 1. U_T steadily increased on enlarging the ligament and, as expected, went through the origin, i.e. no finite intercept at $l = 0$ was detectable.

In the EWF model, the work to fracture the ligament is dissipated to create the inner and the outer plastic zones. Since such regions grow at different rates, an intercept appears if data are plotted, according to Equation 1, as a function of ligament size. In

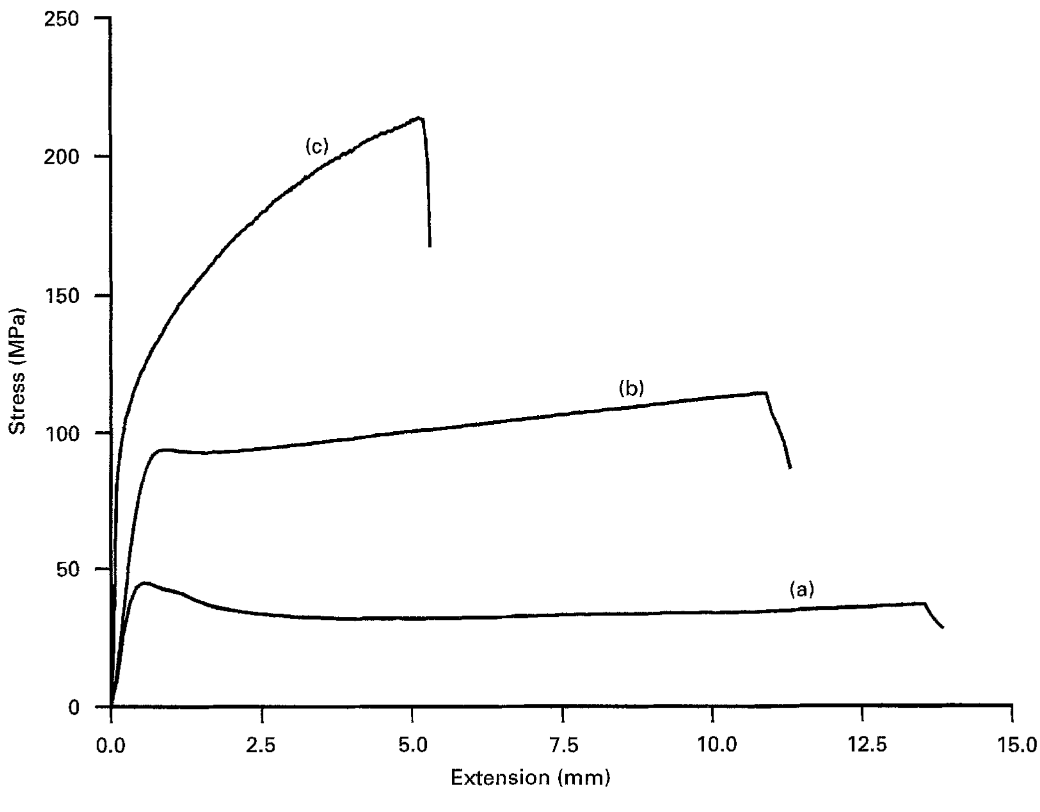

Figure 1 Stress–extension curves for (a) film 1, (b) film 2, and (c) copper.

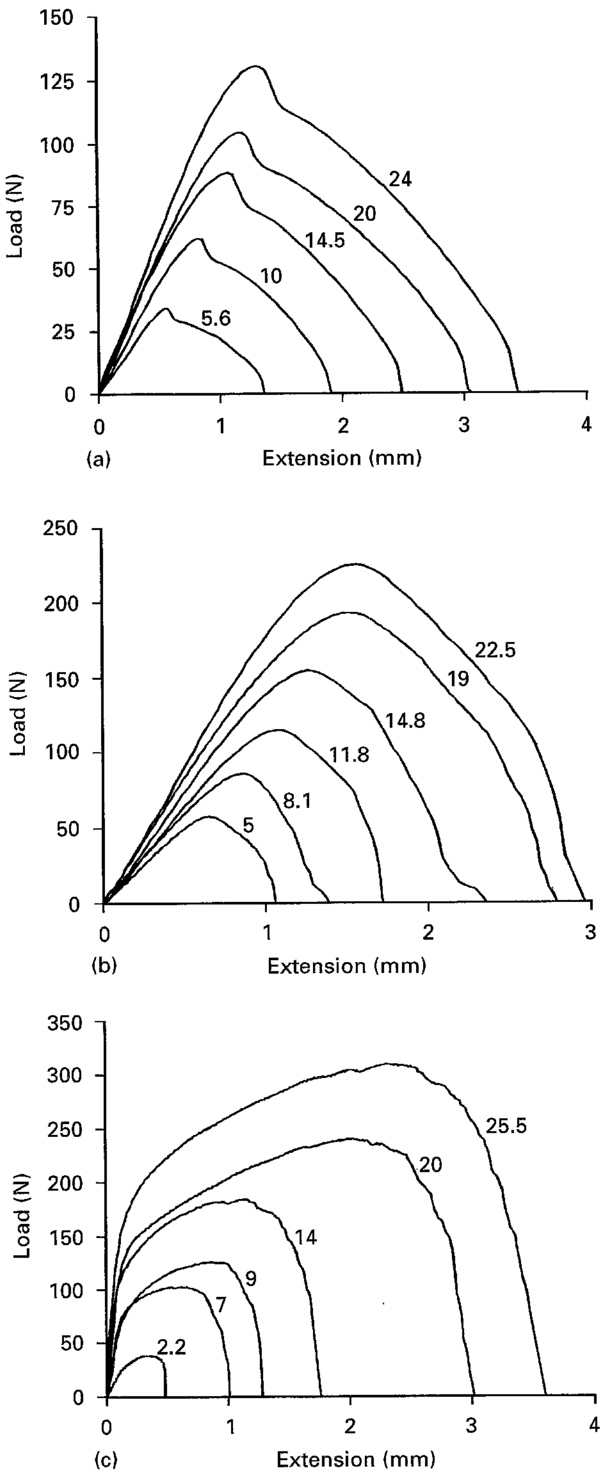

Figure 2 Load–extension curves for selected DENT specimens for (a) film 1, (b) film 2, and (c) copper.

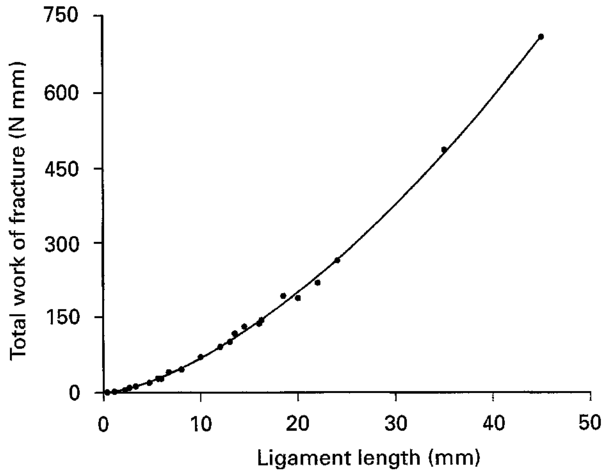

Figure 3 Total work of fracture, as a function of ligament length, for film 1.

a number of papers that have appeared in literature, the ligament size interval investigated is rather limited. If for film 1 we limit the analysis of our data to the interval 5–25 mm, inset in Fig. 4, a straight line can be confidently drawn, with an intercept of $\sim 30\ \mathrm{kJ\,m^{-2}}$; this value would represent the essential term, u_E, for the actual thickness. However, if we consider the complete set of experimental data, which almost spanned two decades of ligament lengths, it is apparent from Fig. 4 that (a) a straight line cannot be fitted to them, and (b) if a straight line is forced through the data, the values of u_E and u_{NE} would depend on the extension of the ligament range considered (the more the interval is extended to the right, the higher the intercept, u_E, and the lesser the slope, βu_{NE}). The EWF model predicts a break in the linearity when the ligament-to-thickness ratio is < 3–5, as a consequence of the prevailing importance of plane strain over plane stress conditions; for the actual case we can include ligaments down to 0.5 mm and yet fulfill the l/t condition. Since no transitions in the state of stress can be conceived (border effects are not dealt with here) doubts could be raised on the capability of Equation 1 to represent data over a large ligament base even in absence of a clear-cut modification of stress conditions.

This situation is not new in fracture mechanics. In J-integral testing frequently the work of propagation, J, does not linearly depend on the advancement of the crack, Δa. It has been then proposed [4, 5] that a power law, of the type $J = A/\Delta a^n$, would give a much better fit. A similar function:

$$u_S = A\,l^n \tag{2}$$

has been utilized to describe specific work data of several polymers [6, 7]. Fig. 4 shows that such a power law could be very accurately fitted to u_S versus l data with a correlation coefficient of 0.994. The importance of this, as first pointed out by Salemi and Nairn [6], is that it is possible to improve the accuracy of measurements. We can utilize reasonably sized ligaments (very small ligaments are difficult to make and to measure precisely, large ligments may cause the film to slip out of the grips because loads are relatively large) to describe u_S curves over a much larger ligment interval not covered experimentally.

A different power law of the type:

$$u_S = A\,l^n + B \tag{3}$$

which would have the advantage of preserving an intercept, B, whose meaning would be the same as for u_E in Equation 1 was also tested. In order to distinguish between Equations 2 and 3 we used samples with very small ligments (down to 0.3 mm). A best fit procedure of Equation 3 to experimental data gave high correlation coefficients but small figures for B so

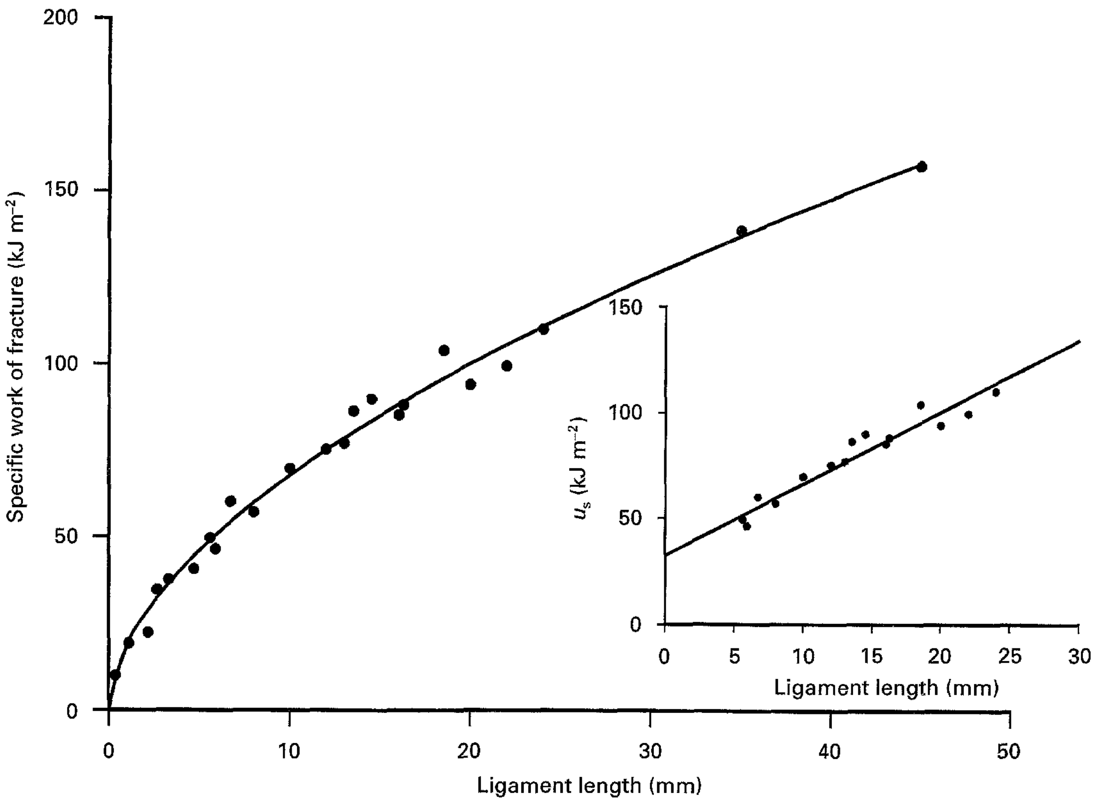

Figure 4 Specific work of fracture, as a function of ligament length, for film 1: line is Equation 2. Inset: linear fit over a restricted ligament interval.

that this term was confidently equated to zero, that is Equation 3 reduced to Equation 2.

A very simple way to check the validity of Equation 2 is to plot it in logarithmic form, that is log(u_S) versus log(l), as shown in Fig. 5. The fact that all points nicely sat on the straight line confirms the effectiveness of Equation 2, irrespective of the ligament length investigated. From the experimentalist's viewpoint this way of representing data is very convenient; in fact, a simple linear regression analysis yields both A and n and a limited number of experimental points may be required.

The same pattern of behaviour was shown by film 2. Fig. 6 shows data in logarithmic form, whereas the inset in the same figure is the conventional linear plot.

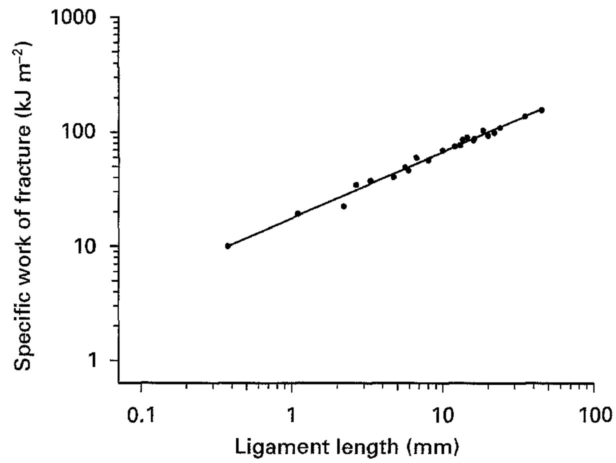

Figure 5 Specific work of fracture, as a function of ligament length, for film 1 in a log–log plot.

3.2. Film 3 (copper)

The EWF method was, since it was proposed, applied to metals [8–12] which greatly differ from polymers in the post-yield behaviour. Most metals markedly strain-harden after yielding, whereas polymers typically strain-soften. Usually metals do not neck after yielding; the deformation spreads away from the site of the initial yield, while in polymers necking occurs in the site of the first yield and the deformation is forced to remain there. In terms of the EWF model this is equivalent to saying that the outer plastic zone is, in metals, much larger than in polymers. To verify whether the differences in the post-yield behaviour, illustrated in Fig. 1, modified the relationship between u_S and l, we tested an annealed film of copper whose strain-hardening behaviour is clear in Fig. 1c. In Fig. 7 data of specific work are represented in double logarithmic form, whereas the usual linear/linear plot is in the inset. It is apparent that also in this case the relationship between u_S and l does not change, i.e. Equation 2 perfectly conforms to experiments.

4. Discussion

The yield stress is not, truly speaking, a materials property for it depends on the level of plastic constraint experienced in the yielding zone. To verify how the state of stress may depend on ligament size we can plot the apparent flow stress (i.e. the maxima on the load–extension curves in Fig. 1) as a function of ligament length. As shown in Fig. 8 the value of σ_{FLOW} when l approached the sample width coincided with

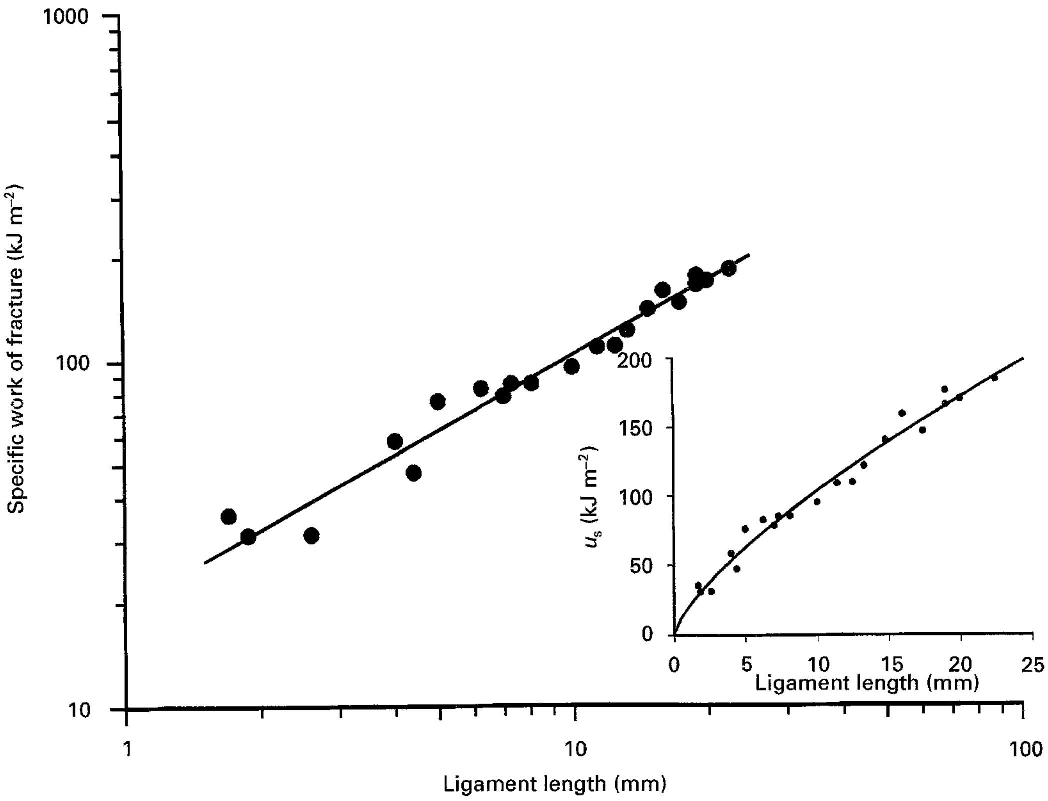

Figure 6 Specific work of fracture, as a function of ligament length, for film 2 in a log–log plot. Inset: same data in a linear plot; line is Equation 2.

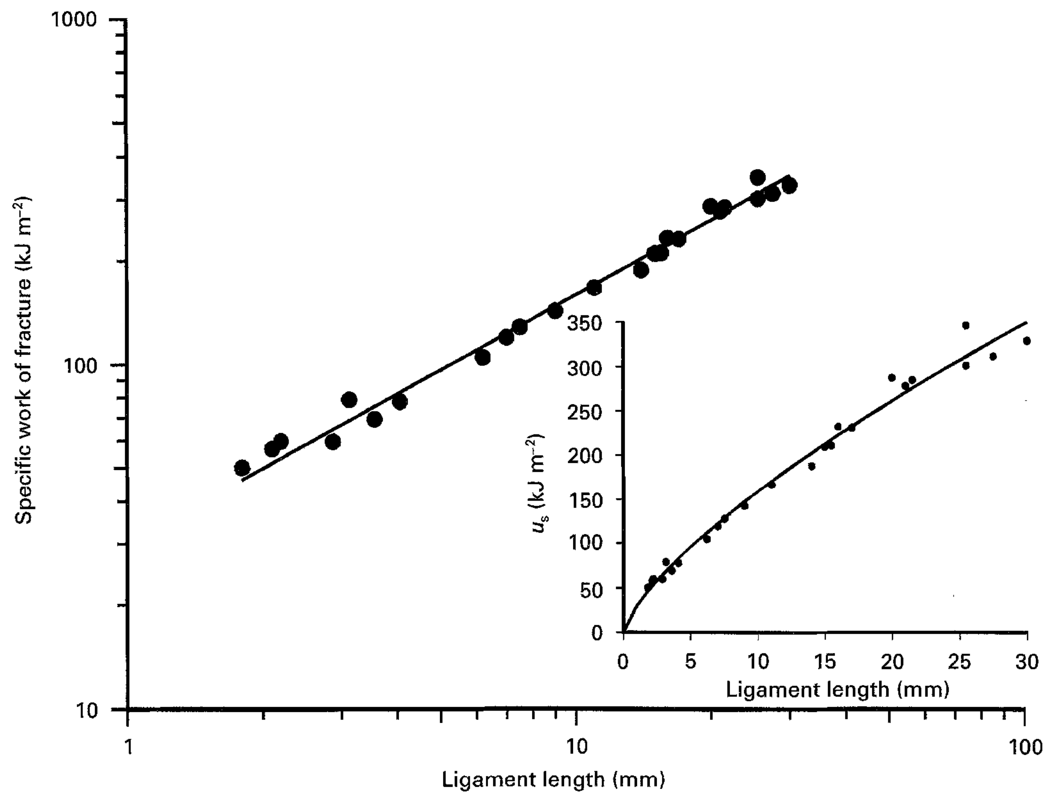

Figure 7 Specific work of fracture, as a function of ligament length, for the copper film in a log–log plot. Inset: same data in a linear plot; line is Equation 2.

the uniaxial yield stress. On diminishing l a certain level of plastic constraint developed that attained a maximum for $l \rightarrow 0$. The maximum constraint was for films 1 and 2 about 1.2. The flow stress was substantially independent of l for ligaments larger than about 10 mm. We could take this value as the lower limit of the ligament interval and apply Equation 1 only to ligaments larger than this value to obtain the true plane-stress essential work of fracture. It is however necessary to define the upper boundary of the

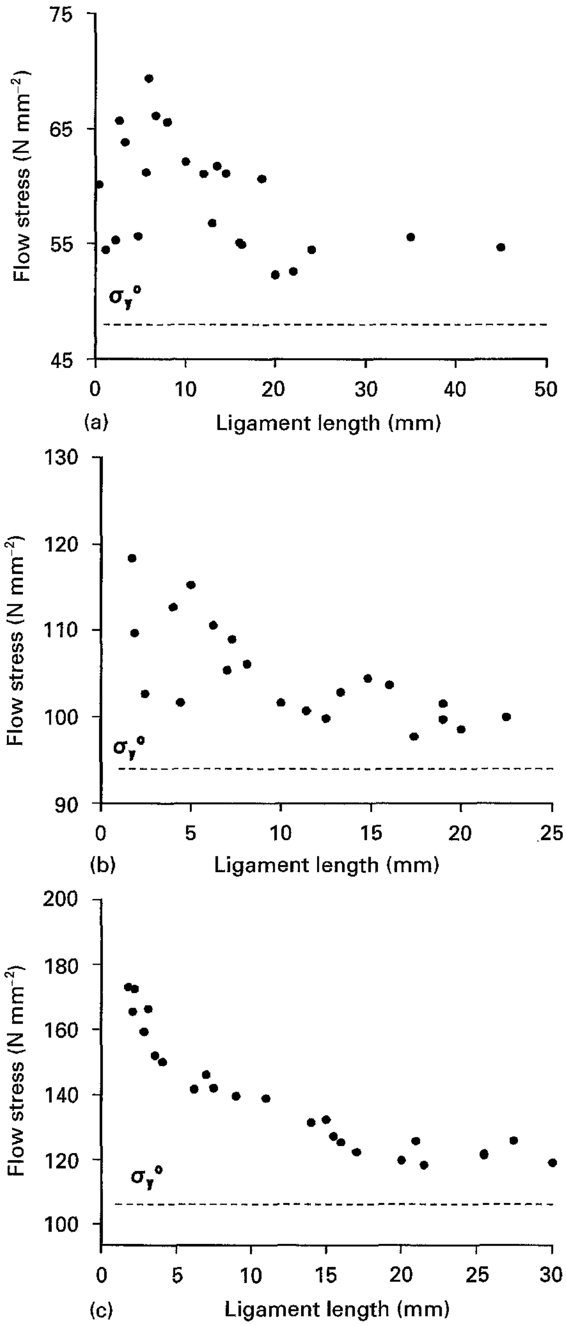

Figure 8 Flow stress, as a function of ligament length, for (a) film 1, (b) film 2, and (c) copper.

ligament interval; this is customarily set at 1/3 of the sample width [13]: 25 mm in the present case. Linear fit in the interval 10–25 mm gave values of about 34 and 35 kJ m^{-2} for film 1 and 2, respectively, see Table I.

In the case of copper the influence of l on σ_{FLOW} was more pronounced, Fig. 8c; the maximum constraint was in fact about 1.7, the same value proposed by Irwin (1.68) [14] as an average over surface and bulk regions in cracked bodies. However, it is admittedly arbitrary to identify the maxima in the load–extension curves, Fig. 2c, with the yield stress since copper, as well as other metals, yields much before reaching the maximum stress (the deformation at yield is in metals about 0.1–0.2%, whereas the deformation at maximum load is, for ductile metals, 20–60%). The 0.2% offset construction to calculate the conventional yield stress was inapplicable to DENT specimens since the

TABLE I Essential work of fracture (linear fit), power law parameters (A and n), H^* (COD) and critical specific work of fracture (from Equation 5)

Material	u_E (kJ m^{-2})	A (kJ m^{-2})/n	H^* (mm)	u_E^* (kJ m^{-2})
Film 1	34.7	19.5/0.72	0.6	13.5
Film 2	35.1	18.5/0.56	0.3	9.4
Copper	71.7	30.2/0.72	0.23	10.5
PEEK 1[a]	62.7	40/0.61	1.97	60.5
PEEK 2[a]	54	33/0.63	1.75	47

[a] From [17].

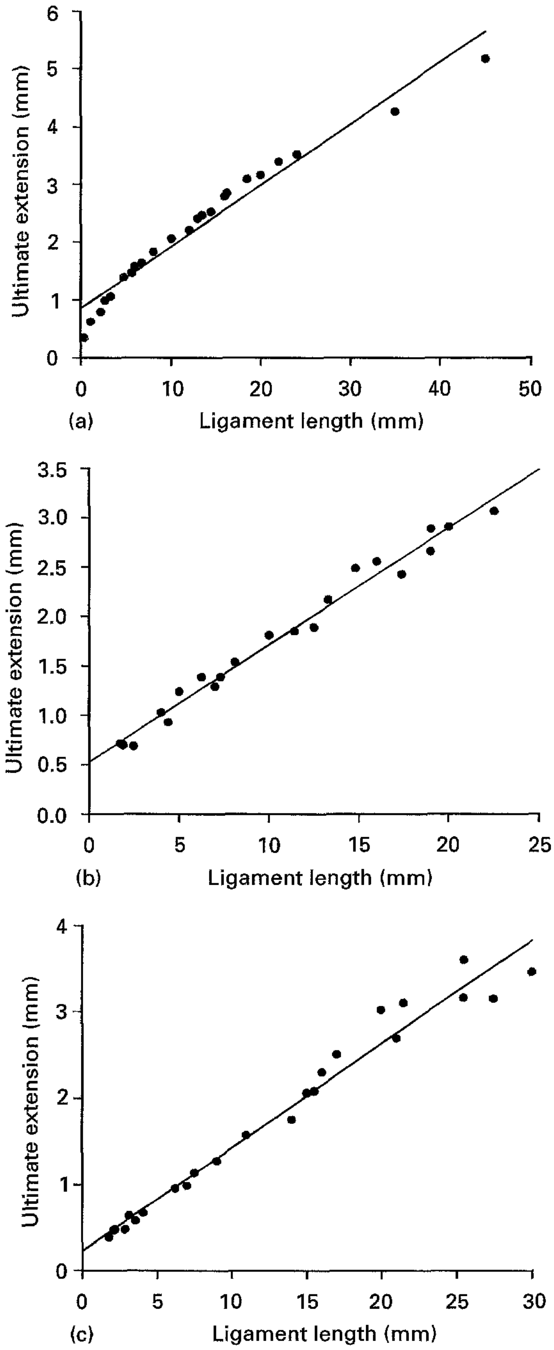

Figure 9 Ultimate extension as a function of ligament length, for (a) film 1, (b) film 2, and (c) copper.

deformation was not evenly distributed along the testpieces. By taking for copper the same ligament interval as for the two polymer films we have for u_E about 72 kJ m^{-2}.

It is apparent that the condition l_{max} = (sample width)/3 implies that the ligament interval to be used changes with the sample width. If the relationship between u_S and l is not linear, the consequence would be that, for the same material, samples of different width would give different values for u_E. In a previous investigation [15] we found that the u_S–l data of PVC samples of dissimilar width, but same thickness, identified a single line which was well described by Equation 2. Linear fits, according to Equation 1, carried out on diverse ligament intervals to satisfy for each width the l_{max} condition, gave different slopes and intercepts. It would be sensible developing a criterion based on Equation 2 in order to find a fracture parameter at least independent of sample width.

By means of the load–extension curves, Fig. 2, it is possible to relate the ultimate elongation, H, of DENT samples to the ligament size as made in Fig. 9 for all materials. A common feature of all plots is the substantially linear relationship between H and l and the existence of a positive intercept, H^*. Several authors [13, 16, 17] have identified H^* with the COD of the material, for the specific thickness investigated. The relationship between u_E and COD is of the type: $u_E = \lambda \sigma_Y^0$ COD, where λ is a constant that may depend on sample geometry and material properties and σ_Y^0 is the uniaxial yield stress. The main drawback with this equation is the fact that the yield stress supposed to describe the local plasticity is the uniaxial value and it has been shown in Fig. 8 that at very small ligaments (H^* is found when $l \rightarrow 0$) the actual yield stress is larger than σ_Y^0. Hashemi and O'Brien [17] used the following relationship to calculate u_E:

$$u_E = 2\gamma \sigma_Y \, \mathrm{COD}/3 \qquad (4)$$

where γ was the fraction of the apparent yield strength of the ligament (P_{max}/lt) to the uniaxial yield stress (σ_Y^0). The COD was taken as the extrapolation to $l = 0$ of the ultimate elongation of single edge notched samples (SENT). The values found for γ were smaller than that proposed by Hill (1.15) [18] for fully yielded DENT samples in plane stress and that found in this work (~ 1) for the same test geometry. However, DENT and SENT samples may present different flow stresses, for the same ligament, since the latter tend to rotate on loading in tension, so that the ligament is not evenly loaded; as a result the apparent flow stress in the SENT geometry is lower than that for the DENT arrangement.

We can tentatively define a critical value of the work of fracture in *quasi*-plane strain conditions by substituting H^* for l in the power law. We thus have:

$$u_E^* = A(H^*)^n \qquad (5)$$

The physical meaning for this is that in ductile materials crack blunting occurs before propagation. If H^* is the vertical displacement at the crack tip, the fictitious advancement of the crack is $H^*/2$. Since there are two facing tips, the minimum ligament required to allow both cracks to blunt should be at least $2(H^*/2)$. No better estimate is at the moment available for the minimum ligament needed to allow blunting so that it has been equated to H^* in Table I in which all relevant parameters of the films so far discussed are collected.

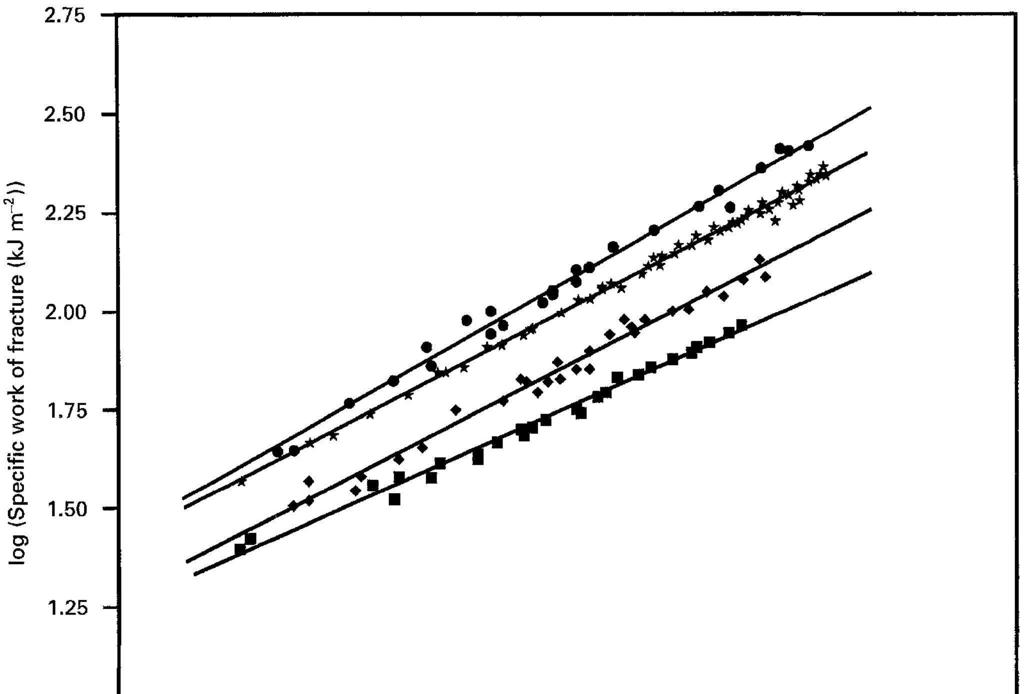

Figure 10 Specific work of fracture, as a function of ligament length, for PVC, Ultranyl, Orgalloy and RT-Nylon (from [6]) of specified thickness in the log–log form. Key: ● Orgalloy (2 mm); ★ RTN 66 (3.175 mm); ◆ PVC (3 mm); ■ Ultranyl (4 mm).

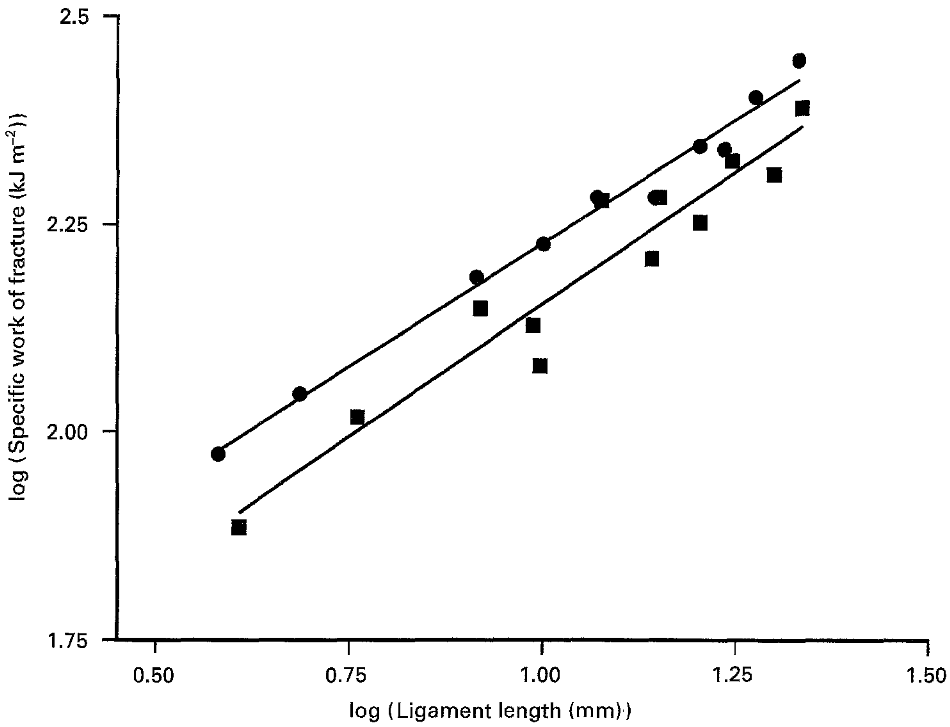

Figure 11 Specific work of fracture, as a function of ligament length, for two films after Hashemi and O'Brian [17] in the log–log form.

We found that Equation 2 complies with experimental data for other materials too and for much thicker samples [15]. We also found that of the two numerical parameters, A and n, that characterized various materials so far examined using the same test geometry here considered, the pre-exponential term, A, showed large variations, while the exponent n was relatively constant, i.e. in log/log plots the u_S–l lines of different materials were substantially parallel, Fig. 10. It is possible that the parameter A more promptly reflects changes in materials properties where as n might be influenced to a larger extent by the test geometry; this point needs to be further investigated. It is worth noting that the log/log plot is a good way to represent data previously analysed according to Equation 1. An example is in Fig. 11 were data taken from Hashemi and O'Brian [17] are reconsidered.

It has to be pointed out that the applicability of Equation 2 is conditioned by the way the ligament fails. It is assumed in the EWF model that once the ligament yields, the crack starts propagating. In brittle polymers failure occurs before yielding and linear fracture mechanics applies. Very ductile polymers behave quite differently. After yielding, extensive drawing of the ligament occurs so that stretching and tearing are the most relevant energy absorbing mechanisms and Equation 2 may not conform with experimental results. In this case it is advisable to adopt other methods to analyse the behaviour of the material such as that used by Hodgkinson and Williams [19].

5. Conclusions

The linear relationship predicted to exist between specific work of fracture, u_S, and the length of the ligament, in DENT specimens of thin films, was not verified; a power law could be fitted to data with very high correlation coefficients. A linear relationship was approximately obeyed if ligaments smaller than about 5–10 mm were neglected; however, the slope of the straight line and, consequently, the value of the intercept, u_E, depended on the upper bound of the ligament range considered. It was found that the post-yield behaviour (strain-hardening or strain-softening) did not influence, at least for the thickness utilized, the fracture behaviour of the materials. In all cases the ultimate elongation had, at vanishing ligaments, a finite value that can be identified with the crack opening displacement; this value can be tentatively taken as the minimum value of the ligament length needed to allow crack blunting before propagation.

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